

UNIT-V

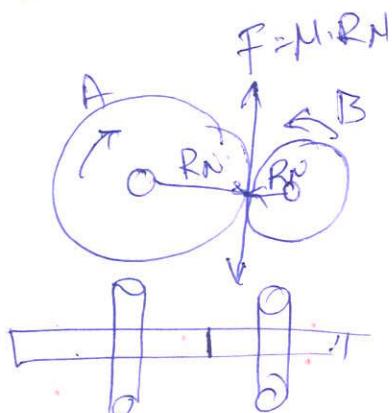
Toothed wheels

The slipping of a belt or rope is a common phenomenon in the transmission of motion or power between two shafts. The effect of slipping is to reduce the velocity ratio of the system. In precision machines, in which a definite velocity ratio is of importance, the only positive drive is by means of gear or toothed wheels.

Friction wheels

The motion and power transmitted by gears is kinematically equivalent to that transmitted by friction wheel or disc. Consider a two plain circular wheels A and B mounted on the shafts, having sufficient rough surfaces and pressing against each other. Let the wheel A be keyed to the rotating shaft and wheel B to the shaft to be rotated.

A friction wheel with the teeth cut on it is known as toothed wheel or gear. Friction wheels can only used for transmission of small powers.



Advantages

- It transmits exact velocity ratio.
- It may be used to transmit large power.
- It has high efficiency.
- It has reliable service.
- It has compact layout.

Disadvantages

- The manufacture of gear requires special tools and equipment.
- The error in cutting teeth may cause vibrations and noise during operation.

Classification of toothed wheel:

1. According to the position of axes of the shafts

- parallel
- Intersecting
- Non-intersecting and non parallel

2. According to peripheral velocity of the gears.

- low velocity
- Medium velocity
- High velocity.

3. According to the type of gearing:-

- a) External gearing
- b) Internal gearing
- c) Rack and pinion.

4. According to the position of teeth on the gear surface.

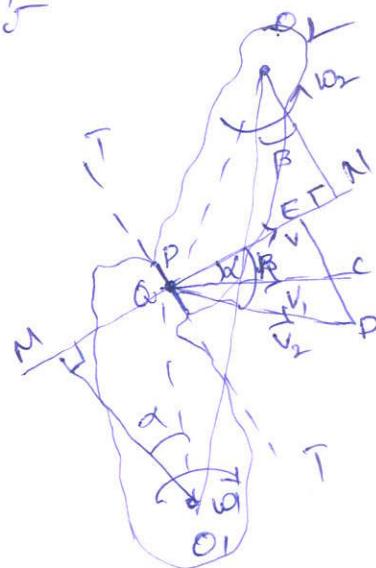
- a) straight
- b) inclined
- c) curved.

Condition for constant velocity Ratio of toothed wheels - Law of Gearing:-

Let TT be the common tangent and MN be the common normal to the curves at the point of contact Q. From the centres O₁ and O₂, draw O₁M and O₂N perpendicular to MN. A little consideration will

Show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2.

Let v₁ and v₂ be the velocities of the point Q on wheels 1 and 2 respectively.



$$v_1 \cos \alpha = v_2 \cos \beta$$

$$(w_1 \times O_1 Q) \cos \alpha = (w_2 \times O_2 Q) \cos \beta$$

$$(w_1 \times O_1 Q) \frac{O_1 M}{O_1 Q} = (w_2 \times O_2 Q) \frac{O_2 N}{O_2 Q}$$

$$(or) w_1 \times O_1 M = w_2 \times O_2 N$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M}$$

from similar triangles $O_1 MP \sim O_2 NP$

$$\frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P}$$

$$\therefore \frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P}$$

This is also equal to

$$\frac{\omega_1}{\omega_2} = \frac{O_2 P}{O_1 P} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

where

D_1 and D_2 are the pitch circle dia.
and T_1 and T_2 are the no. of teeth on
wheels respectively

Velocity of sliding teeth

The velocity of sliding is the velocity of one tooth relative to its mating tooth along the common tangent at the point of contact.

From similar triangles Q_1EC and O_1MQ

$$\frac{EC}{MQ} = \frac{V}{O_1Q} = \omega_1$$

(or)

$$EC = \omega_1 \cdot MQ.$$

Considered as a point on wheel 2,

from similar triangles Q_2CD and O_2NQ

$$\frac{ED}{QN} = \frac{V_2}{O_2Q} = \omega_2$$

or

$$ED = \omega_2 \cdot QN.$$

Let V_s = Velocity of sliding at Q.

$$V_s = ED - EC = \omega_2 \cdot QN - \omega_1 \cdot MQ$$

$$= \omega_2(QP + PN) - \omega_1(MP - QP)$$

$$= (\omega_1 + \omega_2) QP + \omega_2 \cdot PN - \omega_1 \cdot MP$$

Since $\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{PN}{MP}$

(or)

$$\omega_1 \cdot MP = \omega_2 \cdot PN.$$

$$V_s = (\omega_1 + \omega_2) QP.$$

Forms of teeth

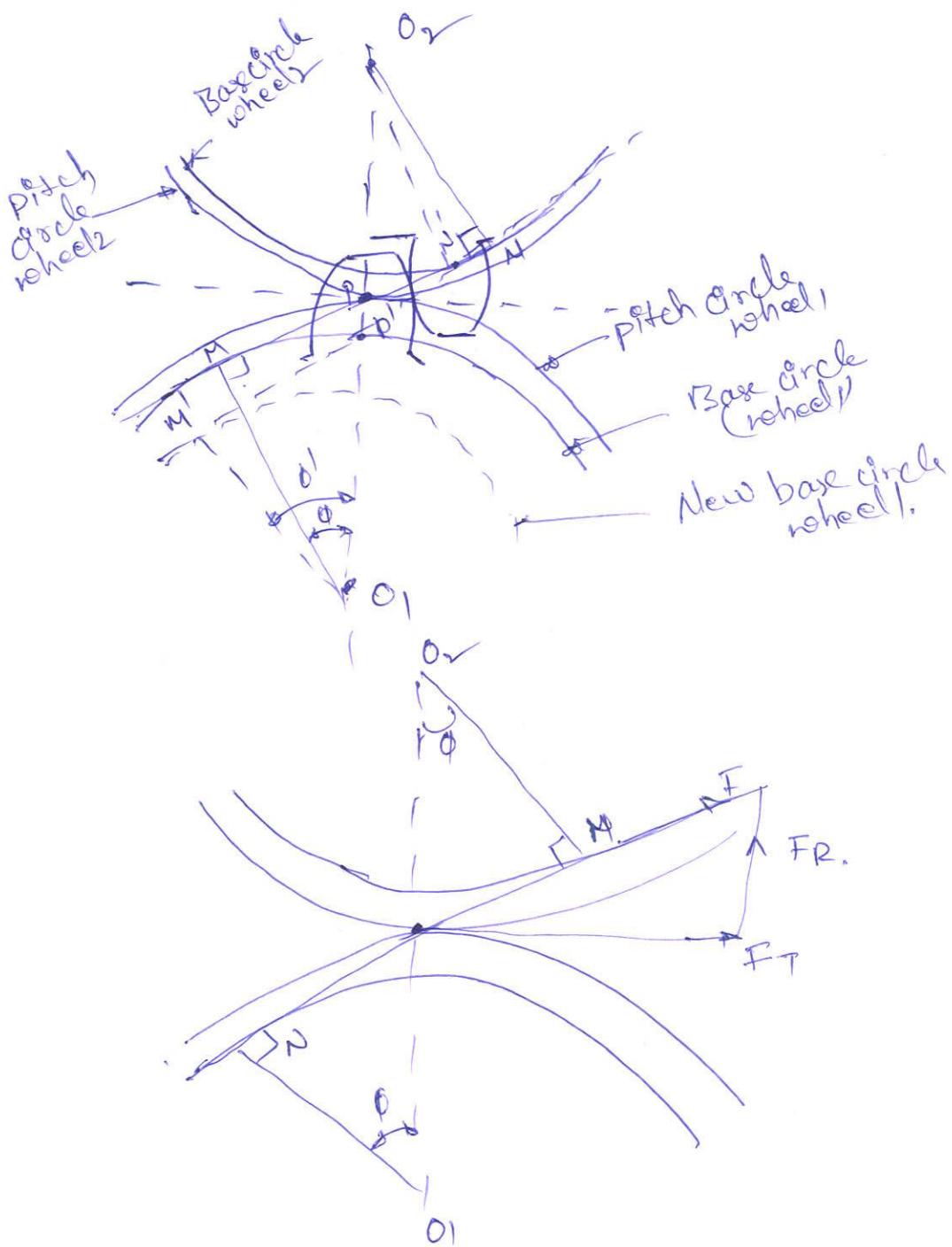
1. Cycloidal teeth
2. Involute teeth

A cycloidal teeth :- A cycloid is a curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line.

When a circle rolls without slipping on the outside of a fixed circle, the curve traced by a point on the circumference of a circle is known as epi-cycloid. On the other hand, if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called hypo-cycloid.

2. Involute Teeth:-

An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which is unwrapped from a reel. Normal In other words, normal at any point of an involute is a tangent to the circle.



From similar triangles O_2NP and O_1MP

$$\frac{O_1M}{O_2N} = \frac{O_1P}{O_2P} = \frac{\omega_2}{\omega_1}$$

$$O_1M = O_1P \cos \theta$$

$$O_2N = O_2P \cos \theta$$

centre distance b/w the base circles

$$O_1 O_2 = O_1 P + O_2 P = \frac{O_1 M}{\cos \phi} + \frac{O_2 N}{\cos \phi}$$
$$= \frac{O_1 M + O_2 N}{\cos \phi},$$

if F is the max. tooth pressure,

then tangential force $F_T = F \cos \phi$

radial or normal force $F_R = F \sin \phi$.

∴ Torque exerted on the gear shaft

$$= F_T \times r.$$

where r = radius of gear of pitch circle.

Effect of altering the centre distance of the velocity ratio for involute teeth gear.

$$\frac{O_1 M}{O_2 N} = \frac{O_1 P}{O_2 P} = \frac{\omega_2}{\omega_1}$$

Similar triangles $O_2 N P$ & $O_1 M P$.

$$\frac{O_1 M}{O_2 N} = \frac{O_1 P}{O_2 P}.$$

Similarly $O_2 N' P'$ and $O_1 M' P'$,

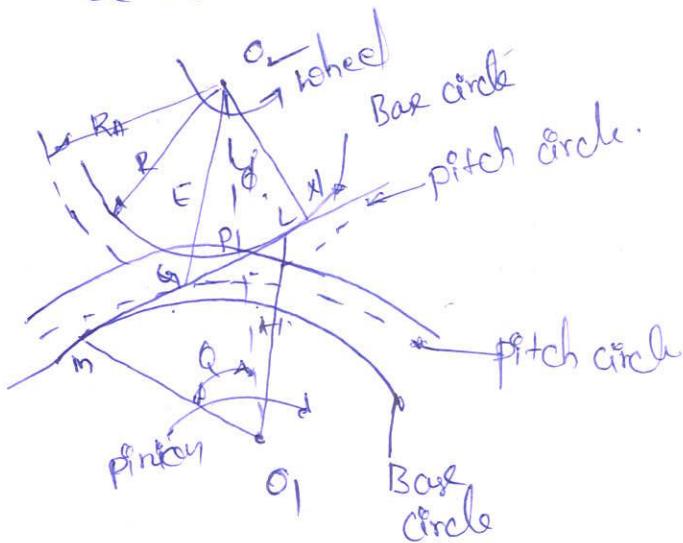
$$\frac{O_1' M'}{O_2 N'} = \frac{O_1' P'}{O_2 P'}$$

But $O_2N = O_2N'$ and $O_1M = O_1'M'$.

$$\therefore \frac{O_1P}{O_2P} = \frac{O_1'P'}{O_2P'}$$

Let T = torque transmitted in N-mm.

Length of path of contact :-



Let $r_A = O_1L$ = Radius of addendum circle of pinion

$R_A = O_2K$ = Radius of addendum circle of wheel,

$r = O_1P$ = Radius of pitch circle of pinion.

$R = O_2P$ = radius of pitch circle of wheel.

we find the base circle of pinion.

$$O_1M = O_1P \cos\phi = r \cos\phi$$

and radius of base circle of wheel

$$O_2N = O_2P \cos\phi = R \cos\phi.$$

Now from right angle triangle O_2KN

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - r^2 \cos^2\phi}$$

$$PN = O_2P \sin\phi = rs \sin\phi.$$

\therefore Length of the part of the path of contact, or the path of approach,

$$KP = KN - PN = \sqrt{(R_A)^2 - r^2 \cos^2\phi} - rs \sin\phi.$$

Similarly from right angle triangle O_1ML

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2 \cos^2\phi}$$

$$MP = O_1P \sin\phi = rs \sin\phi.$$

\therefore Length of part of the path of contact, or path of recess,

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2 \cos^2\phi} - rs \sin\phi.$$

\therefore Length of the path of contact

$$KL = KP + PL = \sqrt{(R_A)^2 - r^2 \cos^2\phi} + \sqrt{(r_A)^2 - r^2 \cos^2\phi} - (r + R) \sin\phi$$

Length of arc of contact

we know that length of the arc of approach

(arc ap)

$$= \frac{\text{length of path of approach}}{\cos \phi} = \frac{KP}{\cos \phi}$$

and the length of the arc of recess (arc ph)

$$= \frac{\text{length of path of recess}}{\cos \phi} = \frac{PL}{\cos \phi}$$

\therefore length of arc of contact GPH is equal to
the sum of the length of arc of approach
and arc of recess.

\therefore length of arc of contact

$$\text{arc GIP} + \text{arc PH} = \frac{KP}{\cos \phi} + \frac{PL}{\cos \phi} = \frac{KL}{\cos \phi}$$

$$= \frac{\text{length of path of contact}}{\cos \phi}$$

Contact Ratio (or Number of pairs of teeth in contact)

$$\text{Contact Ratio} = \frac{\text{length of arc of contact}}{P_c}$$

$P_c = \text{Circular pitch} = \pi m$,

$m = \text{module}$.

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Unit - 5

Gear trains.

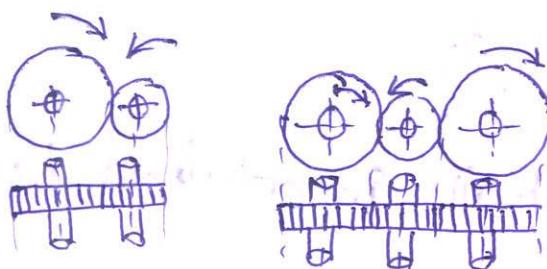
Some times, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such combination is gear train or train of toothed wheels.

Types of Gear trains:

1. Simple gear train
2. Compound gear train
3. Reverted gear train.
4. Epicyclic gear train.

1. Simple Gear-train:

When there is only one gear on each shaft as shown in figure, it is known as simple gear train. The gears are represented by their pitch circle.



N_1 = speed of gear 1 (or driver) in r.p.m

N_2 = speed of gear 2 (follower or driven) in r.p.m

T_1 = No. of teeth on gear 1

T_2 = No. of teeth on gear 2.

$$\therefore \text{speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

Ratio of the speed of the driven to the speed of the driver is known as train value.

Now consider a simple train of gear with one intermediate gear shown in figure

Let N_1 = Speed of driven in r.p.m

N_2 = Speed of intermediate gear in r.p.m

N_3 = Speed of driver in r.p.m

T_1 = No. of teeth on driver

T_2 = No. of teeth on intermediate gear

T_3 = No. of teeth on driven

Driving gear & mesh with intermediate gear,

$$\therefore \text{speed ratio } \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

Similarly intermediate gear mesh with follow

$$\therefore \text{speed ratio } \frac{N_2}{N_3} = \frac{T_3}{T_2}$$

\therefore Speed ratio of gear train is obtained by multiplying the egn

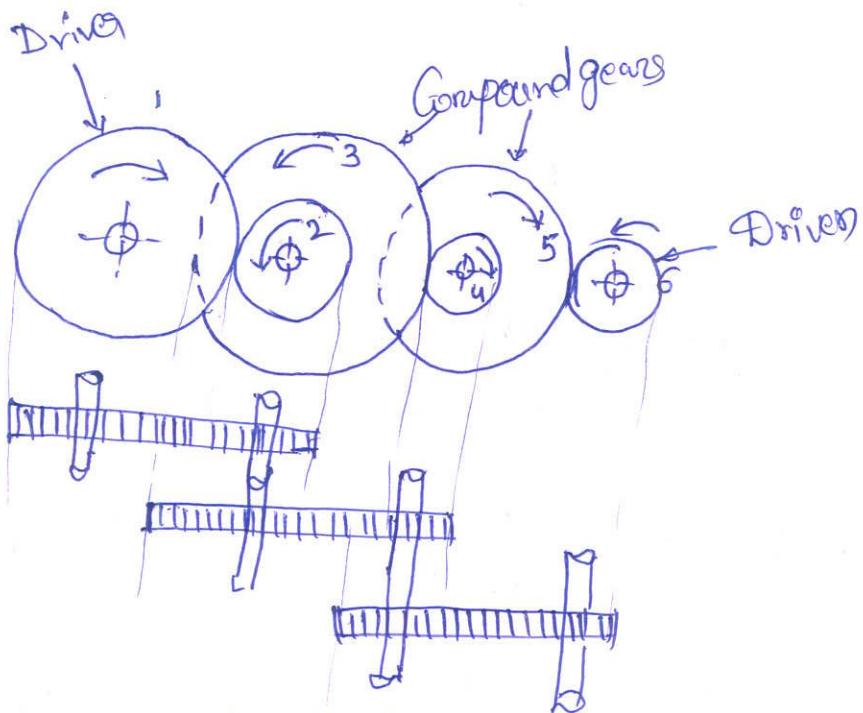
$$\frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} \quad (\text{or}) \quad \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

i.e. Speed ratio = $\frac{\text{speed of driving}}{\text{speed of driven}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$

Train value = $\frac{\text{speed of driven}}{\text{speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$

Compound Geartrain

When there are more than one gear on a shaft, it is called a compound train of gears.



N_1 = Speed of driving gear 1,

T_1 = No. of teeth on driving gear 1

N_2, N_3, \dots, N_6 = Speed of respective gears in r.p.m

T_2, T_3, \dots, T_6 = No. of teeth on respective gears.

Since gear 1 is in mesh with gear 2.

$$\therefore \text{speed ratio } \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

Similarly for gears 3 and gear 4.

$$\text{Speed ratio } \frac{N_3}{N_4} = \frac{T_4}{T_3}$$

and for gear 5 and 6;

$$\text{Speed ratio } = \frac{N_5}{N_6} = \frac{T_6}{T_5}$$

The speed ratio is obtained by multiplying these eqns.

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

$$\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

speed ratio $\frac{\text{speed of the first driver}}{\text{speed of the last driven}}$.

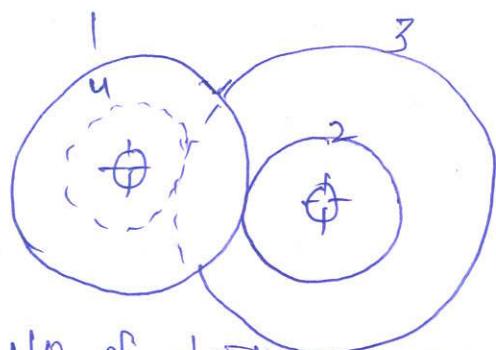
= $\frac{\text{product of no. of teeth on the drivers}}{\text{product of no. of teeth on drivers.}}$

Train value $\neq \frac{\text{speed of the last driven}}{\text{speed of the first driver.}}$

= $\frac{\text{product of the no. of teeth on the drivers}}{\text{product of the no. of teeth on the drivers.}}$

Reversed gear train

When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial - then the gear train is known as reversed gear train.



T_1 = No. of teeth on gear 1

r_1 = pitch circle radius of gear 1

N_1 = Speed of gear 1 in r.p.m.

Similarly :-

T_2, T_3, T_4 = No. of teeth of respective gears,

r_2, r_3, r_4 = Pitch circle radii of respective gears,

N_2, N_3, N_4 = Speed of respective gears in r.p.m.

Since the distance b/w the centres of the shafts of gears 1 and 2 as well as gears 3 and 4 is same,

$$\therefore r_1 + r_2 = r_3 + r_4$$

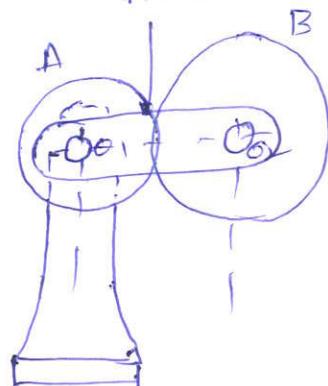
Speed ratio = $\frac{\text{product of no. of teeth on drivers}}{\text{product of no. of teeth on drivers.}}$

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}$$

Epicyclic Gear-trains

In an epicyclic gear-train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in fig., where a gear A and the arm C have a common axis at O, about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O_2 , about which the gears can rotate. If the arm is fixed, the gear-train is simple and gear A can drive gear B or vice-versa. But if gear A is fixed and the arm is rotated about axis of gear A (O_1 -e.g.) then the gear B is forced to rotate upon and around gear A. Such motion is called epicyclic and the gear-trains arranged in such a manner that one or more of their members move upon and around another member are known as epicyclic gear-trains.

Velocity ratio of Epicyclic Gear-train:



1. Tabular Method

2. Algebraic Method

3. Tabular method

T_A = Number of teeth on gear A.

T_B = " " " " gear B.

We know that

$$\frac{N_B}{N_A} = \frac{T_A}{T_B}$$

Since $N_A = 1$ revolution

$$\therefore N_B = \frac{T_A}{T_B}$$

Step 10. Condition of motion

Revolution of elements

Arm C Gear A Gear B.

Arm fixed, gear A rotates

0 +1

$$-\frac{T_A}{T_B}$$

1. through +1 revolution

i.e. 1 rev. Anticlockwise.

2. Arm fixed, gear A rotates
through +x revolution
i.e. x rev. Anti-clockwise

0 +x

$$-\frac{x}{\frac{T_A}{T_B}}$$

3. Add +y revolution to all
elements

+y +y +y

4. total motion

+y x+y $y - x \frac{T_A}{T_B}$

Algebraic Method

Let the arm C be fixed in an epicyclic gear train. The speed of gear A relative to the arm C.

$$= N_A - N_C.$$

Speed of gear B relative to the arm C.

$$= N_B - N_C.$$

Since the gears A and B are meshing directly

$$\therefore \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

Since arm C is fixed,

$$\therefore N_C = 0.$$

$$\frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

If the gear A is fixed then $N_A = 0$

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B}$$

$$\frac{N_B}{N_C} = 1 + \frac{T_A}{T_B},$$

Differential gear of an automobile

The differential gear used in the reardrive of an automobile acts function is

→ To transmit motion from the engine shaft to the rear driving wheels.

→ To rotate the rear wheels at different speeds while the automobile is taking a turn.

As long as the automobile is turning on a straight path, the rear wheels are driven directly by the engine and speed of both the wheels is same. But when the automobile is taking a turn, the outer wheel will run faster than the inner wheel because at that time the outer rear wheel has to cover more distance than the inner rear wheel. This is achieved by epicyclic geartrain with bevel gears as shown in figure.

The bevel gear A is keyed to propeller shaft driven from the engine shaft through universal coupling. This gear A drives the gear B which rotates freely on the axle P. Two equal gears C and D are mounted on two separate parts P and Q of the rear axles respectively. These gears, in turn, mesh with usual pinions E and F which can rotate freely on the spindle provided on the arm attached to gear B.

In hem - the automobile runs on a straight path, the gears C and D must rotate together. These gears are rotated through the spindle on the gear B. The gears E and F do not rotate on the spindle. But when the automobile is taking a turn, the inner rear wheel should have lesser speed than the outer rear wheel and due to relative speed of the inner and outer gears D and C, the gears E and F starts rotating about the spindle axis and at the same time revolve about the axle axis.

Due to this epicyclic effect, the speed of the inner rear wheel decreases by a certain amount and the speed of the outer rear wheel increases by the same amount. This is understood by a table.

step no. Condition of motion

Revolutions of elements

Gear B Gear C Gear E Gear D

1. Gear B fixed - Gear C rotated through α revs (i.e. anticlockwise)	0	+1	$\frac{T_C}{T_E}$	$-\frac{T_E \times T_E}{T_E T_D} = -1$ ($\because T_C = T_D$)
2. Gear B fixed - Gear C rotated through α revolutions	0	$+\alpha$	$\alpha \times \frac{T_C}{T_E}$	$-\alpha$
3. Add α revolutions to all elements	+4	+4	+4	+4
4. total motion	+4	$\alpha + 4$	$4 + \alpha \times \frac{T_C}{T_E}$	$4 - \alpha$

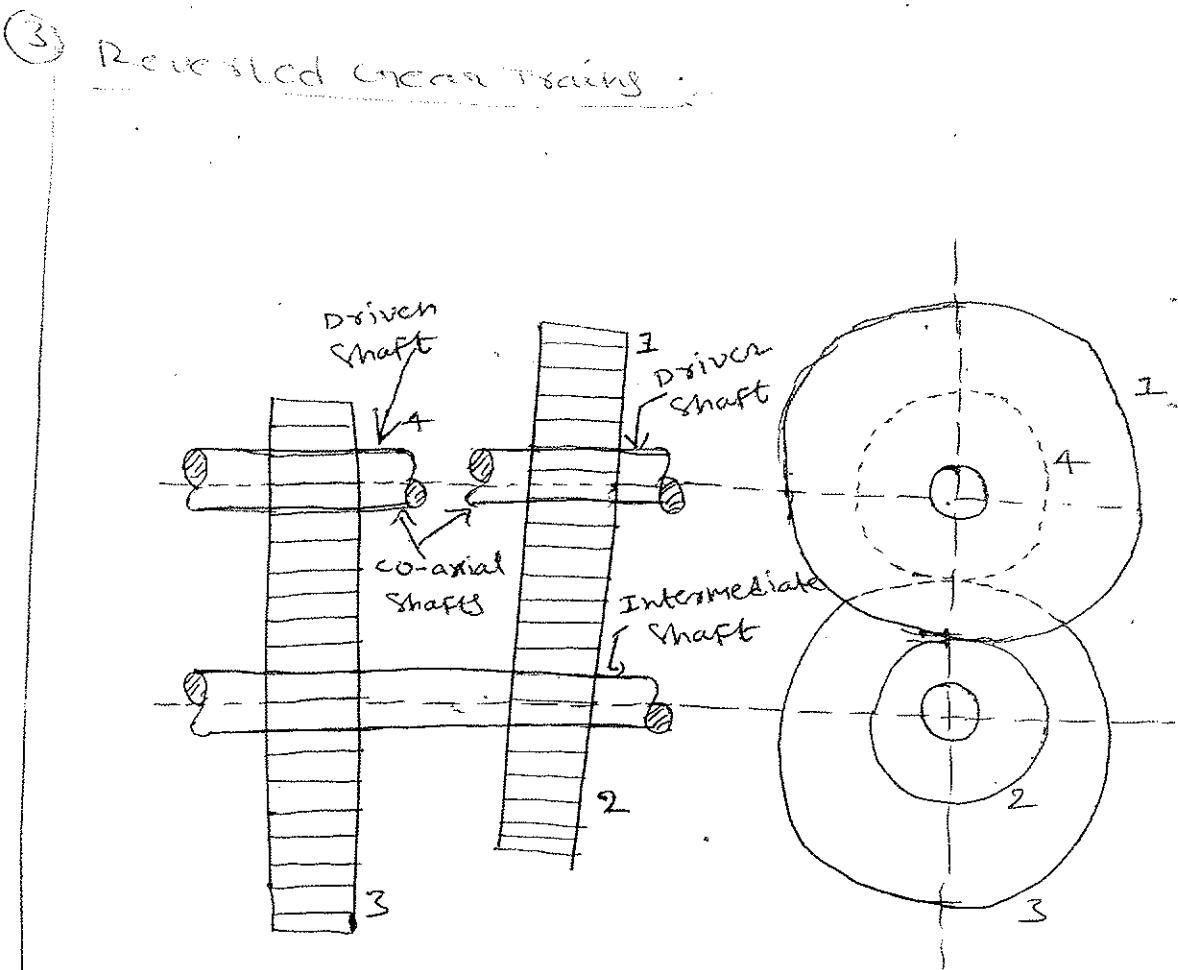
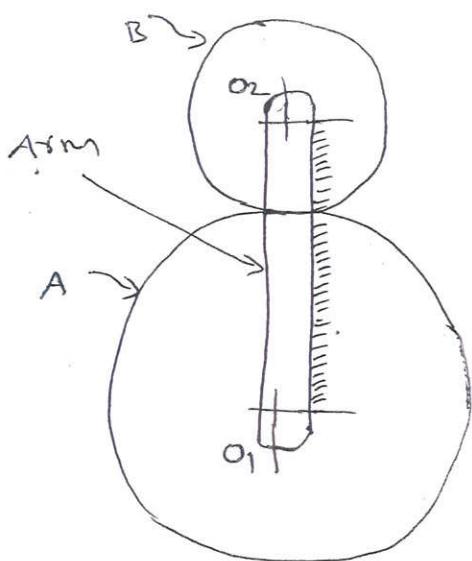


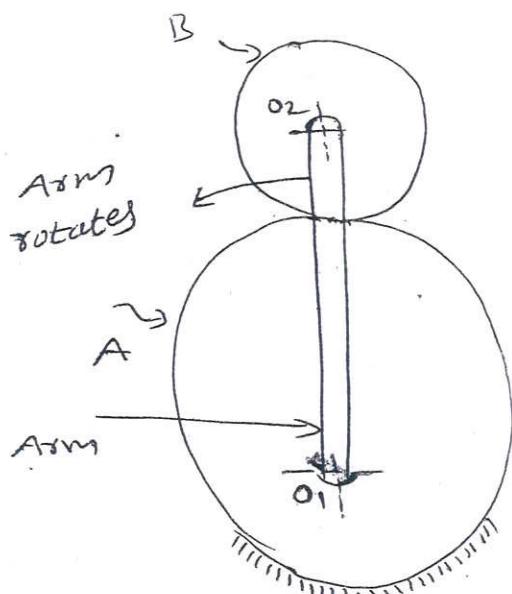
Fig:- Reverted gear train.

when the axes of the driver and driven shafts are co-axial, the compound gear train is known as Reverted gear train or co-axial train as shown in Fig. It should be noted that gears 1 and 4 are on the driver and driven shafts whose axes are collinear. The gears 2 and 3 are connected on the intermediate shaft. In the reverted gear trains shown in Fig above, the gear trains can be only reverted if the sum of pitch radii of gears 1 and 2 equals the sum of pitch radii of gears 3 and 4 (i.e. $r_1 + r_2 = r_3 + r_4$).

Epi-cyclic gear train :-



(a) Simple gear train
(Arm fixed)



(b) Epicyclic gear train
(Gear A fixed).

If the axes of the shafts, over which the gears are mounted are moving relative to a fixed axis, the gear train is known as an epicyclic gear train. Hence in an epicyclic gear train, at least one of the gear axis is in motion relative to the frame. The Fig above shows two gears A and B which are meshing with each other along with an arm. The arm and gear A are having a common axis at O₁ about which they can rotate. The gear B is having its axis at O₂ on the arm. If the arm is fixed, Then the axis of the two shafts on which wheels A and B are mounted will be fixed and we will get a simple gear train as shown in Fig (a). Here gear A can drive gear B or vice versa. But if the wheel A is fixed and arm is rotated about O₁ as shown in Fig (b) then gear B will be rotating about gear A and we get an epicyclic gear train. In this case, the gear (or wheel) B rolls about gear A. Hence the axis of gear B moves relative to the axis of gear A. The epicyclic gear train may be simple or compound.

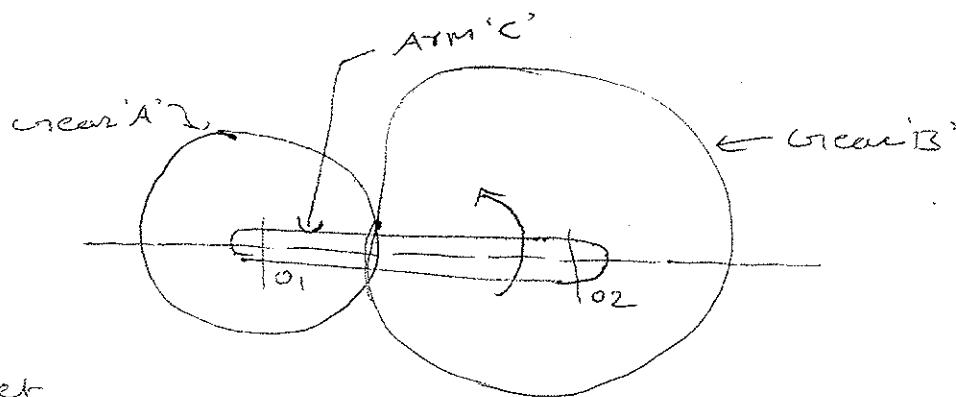
(B) Velocity ratio of Epicyclic gear train :-

The velocity ratio of epicyclic gear train may be determined by various methods. we shall consider only the following two methods.

- (1) Algebraic method (or) Relative velocity method
- (2) Tabular method.

① Algebraic methods :-

The following Fig. Shows two gears A and B and an arm C. Gear 'A' is fixed and the system becomes an example of epicyclic gear train. Let us find the velocity ratio of this epicyclic gear train by Algebraic method.



Let

N_A = Speed of gear 'A' in r.p.m

N_B = Speed of gear 'B' in r.p.m

N_C = Speed of arm 'C' in r.p.m

T_A = No. of teeth on gear 'A'

T_B = No. of teeth on gear 'B'

The speed of gear 'A' relative to arm 'C' = $N_A - N_C$

The speed of gear 'B' relative to arm 'C' = $N_B - N_C$.

Since the gear A and B are meshing directly, therefore they will revolve in opposite directions.

But relative motion between a part of mating gears is always same whether the axes of rotation are fixed (or) moving. And the ratio of relative motion is equal to the inverse ratio of the number of teeth on the gears. Therefore for mating gears 'A' and 'B', we have.

$$\frac{\text{Speed of } 'B' \text{ rel. to arm}}{\text{Speed of } 'A' \text{ rel. to arm}} = -\frac{T_A}{T_B} \quad (-\text{ve sign due to opposite direction})$$

$$\frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B} \quad \textcircled{1}$$

Since wheel 'A' is fixed, hence $N_A = 0$. Therefore the above eqn becomes as

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B}$$

$$-\frac{N_B}{N_C} + 1 = -\frac{T_A}{T_B}$$

$$\frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$

$$N_B = N_C \left[1 + \frac{T_A}{T_B} \right]$$

Hence, if the speed of the arm 'C' is known then speed of wheel 'B' can be calculated. The no. of teeth on gears A and B are already given.

If the instead of gear A, the arm 'C' is fixed, then the ratio of the speed of gear B to speed of gear A is obtained from eqn (i), in which the arm speed N_C becomes zero. Hence From eq (i), we have.

$$\frac{N_B - 0}{N_A - 0} = -\frac{T_A}{T_B}$$

$$\frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

The above eqn shows that when the arm is fixed, the gear train becomes a simple gear train.

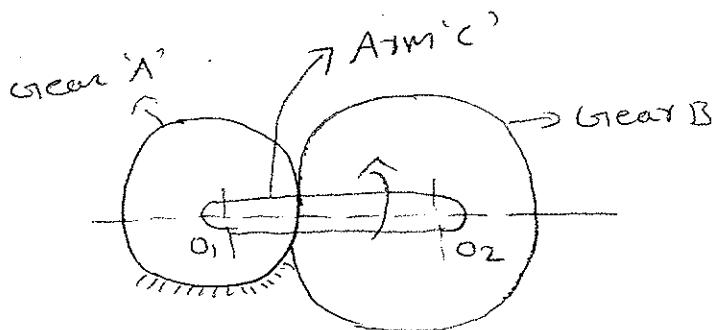
The algebraic method is simple and easy to follow and hence mostly used for solving problems on epicyclic gear trains.

(2) Tabular method :-

In this method, the revolutions of each element (the wheel A, wheel B, and arm C) along with the steps of motion, are written in a tabular form as shown in following table.

Let us apply this method to the epicyclic gear train in following

Fig.



If the arm is supposed to be fixed, then the axes of both the wheels are also fixed relative to each other.

Let the wheel 'A' is rotated by one revolution anti-clock wise, then the wheel 'B' will make $\frac{TA}{TB}$ revolutions clockwise.

This is because $\frac{w_B}{n_A} = \frac{TA}{TB}$ and $n_A = 1$ revolution, Then $n_B = \frac{TA}{TB}$.

If anti clockwise rotation is assumed positive and clockwise rotation as negative then wheel A makes +1 revolution whereas wheel B makes $-\frac{TA}{TB}$ revolution. This relative

Motion is shown in the first row of the table.

It also means that if the wheel A makes $+x$ revolutions, then the wheel B will make $-x \frac{TA}{TB}$ revolution. This statement is shown in the second row of the table.

The second row is obtained by multiplying x to the first row of the table.

Now give each element of epicyclic train + y revolution. This means that whole system is bodily swivelled by y revolution. This statement is shown in third row. The result motion of each element of the gear is obtained by adding up the second and third rows. This statement is shown in fourth row.

S.no	Steps of motion	Revolution of elements		
		Arm C	wheel A'	wheel B
1	Arm is fixed, rotate A through +1 revolution i.e. 1 revolution Anti clockwise	0	+1	$-\frac{T_A}{T_B}$
2	Arm is fixed. Rotate A through +x revolutions.	0	+x	$-x \frac{T_A}{T_B}$
3	All +y revolution to all elements	+y	+y	+y
4	Resultant motion (i.e. adding second & third row)	+y	x+y	$-x \frac{T_A}{T_B} + y$
		Fifth column	Second column	Third column

In the third column of revolution of elements of the Fourth row, there are four unknowns i.e. x, y, T_A and T_B . If two conditions of rotation of any two elements are known, then speed of third element may be obtained by substituting the given data in the third column of the Fourth row.

PROBLEMS

- Q) The arm, of an epicyclic gear train rotates at 100 r.p.m. in the anti-clockwise direction. The arm carries two wheels A and B having 36 and 45 teeth respectively. The wheel 'A' is fixed and the arm rotates about the centre of wheel A. Find the speed of wheel B, what will be the speed of 'B', if the wheel A instead of being fixed, makes 200 r.p.m. clockwise? Solve by both the methods i.e algebraic method and tabular method.

SOL:- Given:-

Arm speed, $N_C = 100 \text{ r.p.m. (Anticlock wise)}$

Anticlockwise rotation is taken +ve whereas clockwise rotation as -ve.

No. of teeth on A, $T_A = 36$

No. of teeth on B, $T_B = 45$.

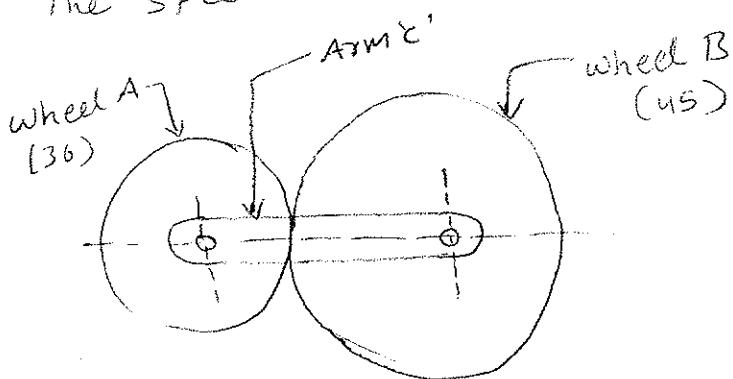
(i) wheel A is fixed. Hence $N_A = 0$, Find N_B .

(ii) wheel A makes 200 r.p.m. clockwise, Find N_B .

(a) Solution by Algebraic Method :-

The speed of wheel A relative to arm C = $N_A - N_C$

The speed of wheel B relative to arm C = $N_B - N_C$.



$$\frac{\text{Speed of } B \text{ relative to arm } C}{\text{Speed of } A \text{ relative to arm } C} = \frac{-T_A}{T_B} \quad \left[\begin{array}{l} \text{(-ve sign is due to rotation} \\ \text{in opposite direction.)} \end{array} \right]$$

$$\frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

(i)

(i) Speed of wheel B when A is fixed

Hence $n_A = 0$, but speed of arm C, $n_C = +100 \text{ r.p.m}$

$$T_A = 36, T_B = 45.$$

Substituting these values in eqn(i), we get

$$\frac{n_B - 100}{0 - 100} = \frac{-36}{45} = -\frac{4}{5}$$

$$n_B - 100 = -100 \left(-\frac{4}{5} \right)$$

$$n_B - 100 = 80$$

$$\therefore n_B = 100 + 80 = 180 \text{ r.p.m. anticlockwise. } \underline{\text{Ans}}$$

[\because +ve value means the direction is anticlockwise]

(ii) Speed of wheel B when A makes 200 r.p.m. clockwise.
[$n_A = -200 \text{ r.p.m. } (-\text{ve sign due to clockwise rotation})$]

Here $n_A = -200 \text{ r.p.m. } (-\text{ve sign due to clockwise rotation})$

$$n_C = +100$$

Substituting these values in eqn (i), we get

$$\frac{n_B - 100}{-200 - 100} = \frac{-36}{45} = -\frac{4}{5}$$

$$n_B - 100 = -300 \left[-\frac{4}{5} \right] = 240$$

$$n_B = 240 + 100 = 340 \text{ r.p.m. anticlockwise. } \underline{\text{Ans}}$$

b) Solution by Tabular Method :-

S.No.	Steps of motion	Resolution of elements		
		Arm C	Wheel A	Wheel B
1.	Arm is fixed. rotate A through +1 revolution (i.e 1 revolution anti-clockwise)	0	+1	$-T_A/T_B$
2	Arm is fixed. rotate wheel A through $+x$. revolutions (multiplied by x to all)	0	$+x$	$-x T_A/T_B$
3	Add $+y$ revolution to all elements	$+y$	$+y$	$+y$
4	Resultant motion (i.e add second & third row)	$+y$	$x+y$	$-x T_A/T_B + y$

Speed of wheel 'A' when A is fixed
Speed of arm is $100\text{ r.p.m. anti-clockwise}$. But from table
The resultant motion of the arm is y . Hence
 $y = +100\text{ r.p.m.}$

As wheel A is fixed, hence $n_A = 0$. But From the table above
The resultant motion of wheel A is $(n+y)$.

$$n+y = 0 \quad \text{or} \quad n = -y = -100\text{ r.p.m.}$$

From the table, The resultant motion of wheel B is $-n \frac{T_A}{T_B} + y$

$$\begin{aligned}\therefore \text{Speed of wheel B} &= -n \frac{T_A}{T_B} + y \\ &= -(-100) \frac{36}{45} + 100 \quad (\because n = -100 \text{ r.p.m.}) \\ &= 80 + 100 = 180\text{ r.p.m.} \quad (\text{anti-clockwise})\end{aligned}$$

Ans

(ii) Speed of wheel 'B' when A makes $200\text{ r.p.m. clockwise}$.

Here $n_A = -200\text{ r.p.m.}$ (-ve sign due to clockwise motion)

But From table, The resultant motion of wheel A is $(n+y)$. Hence

$$n+y = -200 \quad \text{(i)}$$

Arm speed is $100\text{ r.p.m. anti-clockwise}$. But arm speed from the table is y .

$$y = +100$$

Substituting the value of y in eqn(i) we get

$$n = -200-y = -200-100 = -300$$

Now From the table, The resultant motion of wheel 'B'

$$\text{is } -n \frac{T_A}{T_B} + y$$

$$\therefore \text{Speed of wheel B} = -n \frac{T_A}{T_B} + y$$

$$\begin{aligned}
 \therefore \text{Speed of wheel } n &= -n \frac{T_A}{T_B} + y \\
 &= -(-300) \times \frac{36}{45} + 100 \\
 &= 240 + 100 \\
 &= 340 \text{ r.p.m. anticlockwise } \underline{\text{Ans}}
 \end{aligned}$$

('cause positive value means, that the direction of rotation
is anticlockwise.)

Differential gear of an automobile.

The differential gear used in the rear drive of an automobile is shown in Fig. Its function is.

- To transmit motion from engine shaft to the rear driving wheel, and
- To rotate the rear wheels at different speeds while the automobile is taking a turn.

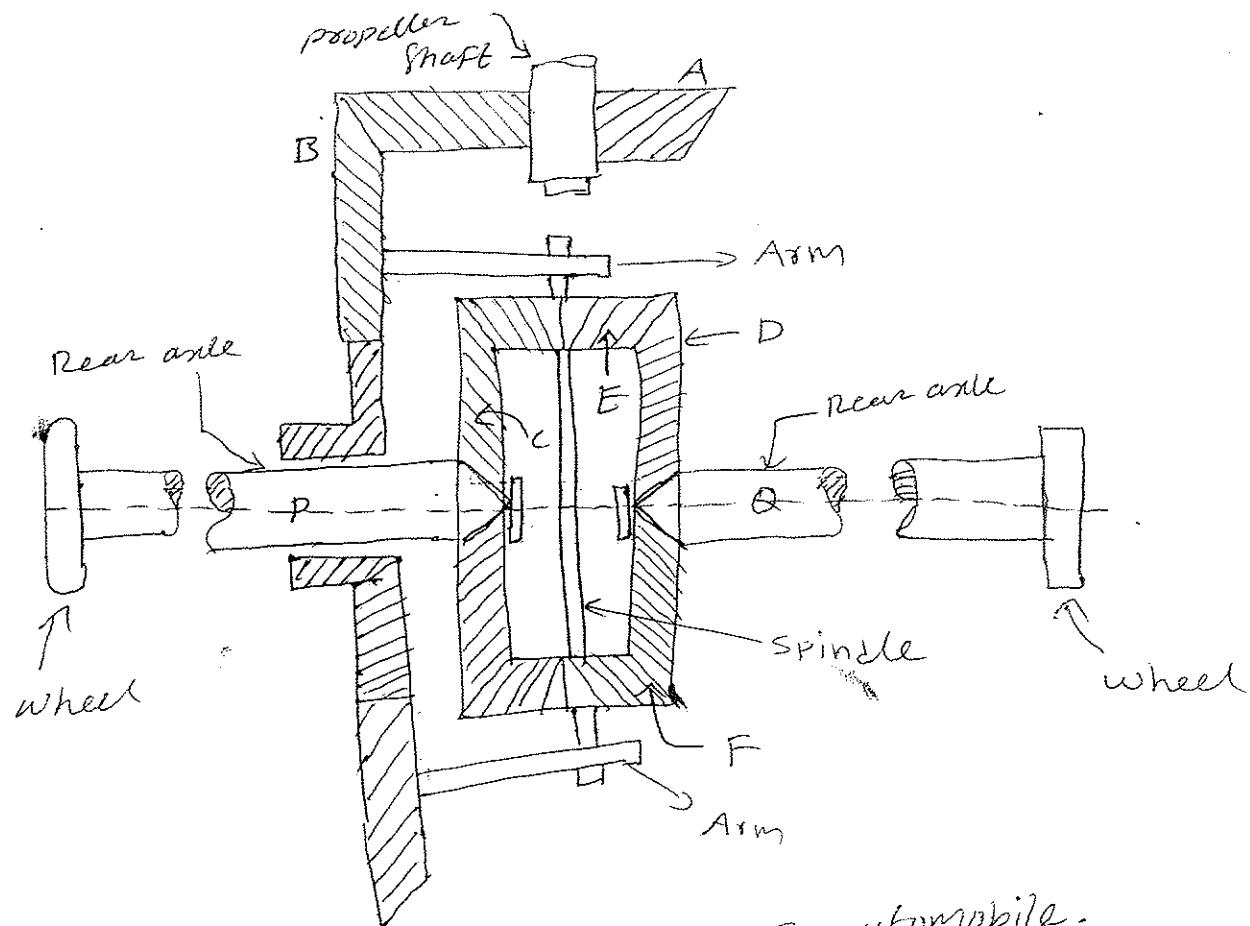


Fig:- Differential gear of automobile.

As long as the automobile is running on a straight path, the rear wheel are driven directly by the engine and speed of both the wheel is same. But when the automobile is taking a turn, the outer wheel will run faster than the inner wheel because at that time the outer rear wheel has to cover more distance than the inner wheel.

This is achieved by epicyclic gear train with bevel gears as shown in Fig.

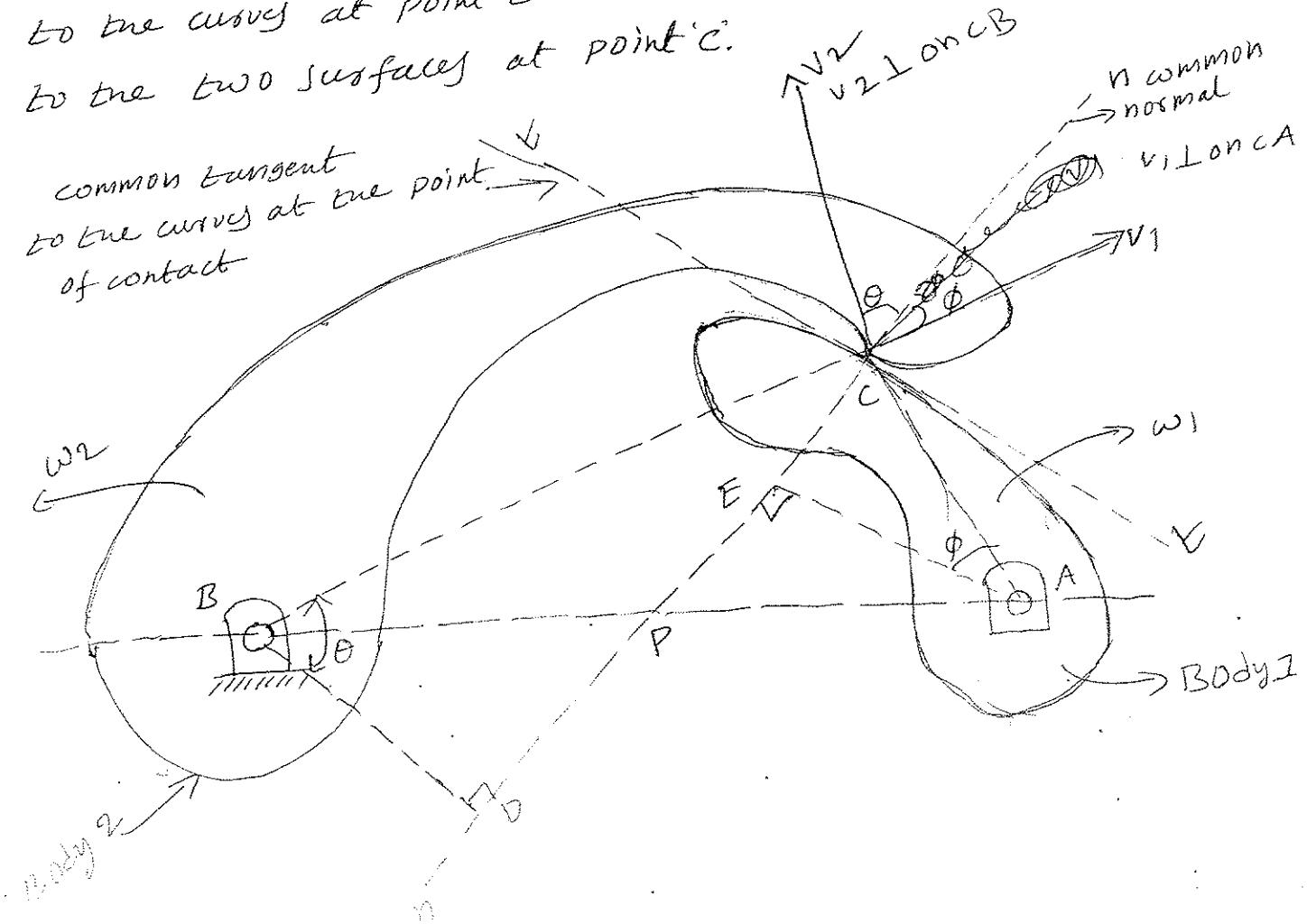
Due to this epicyclic effect, the speed of the inner gear wheel decreases by a certain amount and the speed of the outer gear wheel increases, by the same amount. These may be well understood by drawing the table of motion as follows.

S.NO	condition of motion	revolution of elements.			
		Gear B	Gear C	Gear E	Gear D
1	Gear B-Fixed - Gear C rotated through +1 revolution (i.e 1 revolution anti-clock wise)	0	+1	$+\frac{T_C}{T_E}$	$-\frac{T_C}{T_E} \times \frac{T_E}{T_D} = -1$ ($\because T_C = T_D$)
2	Gear B Fixed - Gear C rotated through $+x$ revolution	0	$+x$	$x \times \frac{T_C}{T_E}$	$-x$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	TOTAL motion	y	$x+y$	$y+x\frac{T_C}{T_D}$	$y-x$

* Law of Gearing or condition for constant velocity ratio of gear wheels :-

Law of Gearing states that the common normal to the two surfaces at the point of contact intersects the line joining the centre of rotation of two surfaces at a fixed point, which divides the centre distance inversely as the ratio of angular velocity.

Let the two curved bodies 1 and 2 are rotating about the centre A and B as shown in fig. below. The two bodies are in contact at point C. The body 1 is rotating clockwise with angular velocity ω_1 and the body 2 is rotating anticlockwise with angular velocity ω_2 . Let E-E is the common tangent to the curves at point C and N-N is the common normal to the two surfaces at point C.



Let

v_1 = linear velocity of point C when point C is assumed to be on the surface of the body 1. This velocity will be \perp to the line AC and will be equal to $w_1 * AC$.

v_2 = linear velocity of point C when point 'C' is assumed to be on the surface of the body 2. This velocity will be \perp to the line BC and will be equal to $w_2 * BC$.

Join the two centre A and B which cuts the common normal at point 'P'

Let θ = angle made by ' v_1 ' with common normal n-n

ϕ = angle made by ' v_1 ' with normal n-n.

From 'B' draw $BD \perp$ on n-n and from A draw $AE \perp$ on n-n.

As v_2 is perpendicular to BC hence $\angle BCA v_2 = 90^\circ$, now

$$\angle DCB = 180 - \angle BCA v_2 - \theta = 180 - 90 - \theta = 90 - \theta$$

Now as v_1 is perpendicular to AC, hence $\angle CAE = 90^\circ$, now

$$\angle ACE = 180 - \phi - \angle CAE = 180 - \phi - 90^\circ = 90^\circ - \phi$$

In $\triangle BDC$ $\angle DCB = 90 - \theta$ $\therefore \angle DBC = \cancel{90}^\circ - \theta$

In $\triangle AEC$, $\angle ACE = 90 - \phi$ $\therefore \angle CAE = \phi$.

If the two surfaces are remain in contact, one surface may slide relative to the other along the common tangent t-t. But the relative motion between the surfaces along the common normal n-n must be zero to avoid separation of the two surfaces or the penetration of the two surfaces into each other.

Moving linear motion of revolution. If the movement is pure rolling or sliding.

The DS BDP and AEP are similar

$$\frac{BP}{AP} = \frac{DP}{PE}$$

Substituting the value of $\frac{BP}{AP}$ in eq above (1),
we get

$$\frac{\omega_1}{\omega_2} = \frac{DP}{PE}$$

$$\omega_1 \times PE = \omega_2 \times DP \rightarrow (2)$$

velocity of sliding :-

If two surfaces are to remain in contact, one surface may slide relative to the other along the common tangent E-E of fig. above.

The velocity of sliding is the velocity of one surface relative to the other surface along the common tangent at the point of contact, ref the fig. above.

component of v_1 along E-E = $v_1 \sin \theta$

component of v_2 along E-E = $-v_2 \sin \theta$ [-ve sign due to opposite direction.]

\therefore velocity of sliding = relative velocity b/w two surfaces
along E-E

$$= v_1 \sin \theta - (-v_2 \sin \theta)$$

$$= v_1 \sin \theta + v_2 \sin \theta$$

$$= (\omega_1 \times AC) \sin \theta + (\omega_2 \times BC) \times \sin \theta$$

$$(\because v_1 = \omega_1 \times AC \quad \{-v_2 = \omega_2 \times BC\})$$

$$= (\omega_1 \times AC) \frac{EC}{AC} + (\omega_2 \times BC) \times \frac{DC}{BC}$$

(as $AC \sin \theta = EC$ & $BC \sin \theta = DC$)

component of v_1 along normal $n-n = v_1 \cos\phi$

component of v_2 along normal $n-n = v_2 \cos\theta$

\therefore relative motion along normal $n-n = v_1 \cos\phi - v_2 \cos\theta$

For proper contact,

relative motion along normal = zero

$$v_1 \cos\phi - v_2 \cos\theta = 0$$

$$v_1 \cos\phi = v_2 \cos\theta$$

$$(\omega_1 \times AC) \cos\phi = (\omega_2 \times BC) \times \cos\theta$$

$$(\omega_1 \times AC) \times \frac{AE}{AC} = (\omega_2 \times BC) \times \frac{BD}{BC}$$

(\because in $\triangle ACE$, $\cos\phi = \frac{AE}{AC}$ and in $\triangle BDC$,
 $\cos\theta = \frac{BD}{BC}$)

$$\omega_1 \times AE = \omega_2 \times BD$$

$$\frac{\omega_1}{\omega_2} = \frac{BD}{AE}$$

$$\frac{\omega_1}{\omega_2} = \frac{BP}{AP} \quad (\because \Delta BDP \text{ and } AEP \text{ are similar,} \\ \text{Hence } \frac{BD}{AE} = \frac{BP}{AP})$$

The above equation shows that the common normal to the two surfaces at the point of contact divides the line joining the centres of rotation in the inverse ratio of the angular velocities. But this ratio angular velocities must be constant for all positions of wheel. This will be, if the point P is a fixed point.

Thus for constant angular velocity ratio of the two surfaces, the common normal at the point of contact must pass through the pitch point (fixed point) on the line

$$= \omega_1 \times EC + \omega_2 \times PC$$

$$= \omega_1 \times (PC - PE) + \omega_2 \times (DP + PC)$$

($\because EC = PC - PE$ and $DC = DP + PC$)

$$= \omega_1 \times PC - \omega_1 \times PE + \omega_2 \times DP + \omega_2 \times PC$$

$$= \omega_1 \times PC + \omega_2 \times PC \quad (\because \text{From eqn } ② \omega_1 \times PE = \omega_2 \times DP)$$

$$= (\omega_1 + \omega_2) PC$$

The above can shows that the velocity of sliding is equal to the product of sum of angular velocities and the distance from the point of contact to the point of intersection of the common normal and the line joining the centre of rotation (i.e. pitch point.)

- ① The following Fig shows a differential gear used in a motor car. The pinion A on the propeller shaft has 12 teeth and gears with the crown gear B which has 60 teeth. The shafts P and Q form the rear axle to which the road wheels are attached. If the propeller shaft rotates at 1000 r.p.m. and the road wheel attached to axle Q has a speed of 210 r.p.m., while taking a turn, find the speed of road wheel attached to axle P.

Sol. - Given: $T_A = 12, T_B = 60$

$$N_A = 1000 \text{ r.p.m.}, N_Q = N_D = 210 \text{ r.p.m.}$$

Since propeller shaft or the pinion A rotates at 1000 r.p.m., therefore speed of crown gear B.

$$N_B = N_A \times \frac{T_A}{T_B} = 1000 \times \frac{12}{60}$$

$$N_B = 200 \text{ r.p.m.}$$

The Table of motion is given below.

S.NO	condition of motion	revolution of elements			
		gear B	inertia	gear C	gear D
1	Gear B fixed - gear C rotated through +1 revolution (i.e. π rev. anticlock wise)	0	+1	$\frac{+T_C}{T_E}$	$-\frac{T_C \times T_E}{T_E \times T_D} = -1$ $(\because T_C = T_D)$
2	Gear B fixed - gear C rotated through + π rev.	0	+ π	$+2 \times \frac{T_C}{T_E}$	- π
3	Add $+y$ rev. to all element	y	$\textcircled{1} y$	$\frac{T_C}{T_E} + y$	$+ \textcircled{2} y$
4	Total motion	+y		$y + \frac{T_C}{T_E}$	$y - \pi$

Since the speed of gear 'B' is 200 r.p.m, therefore from 3rd.
Fourth row of the table

$$y = 200$$

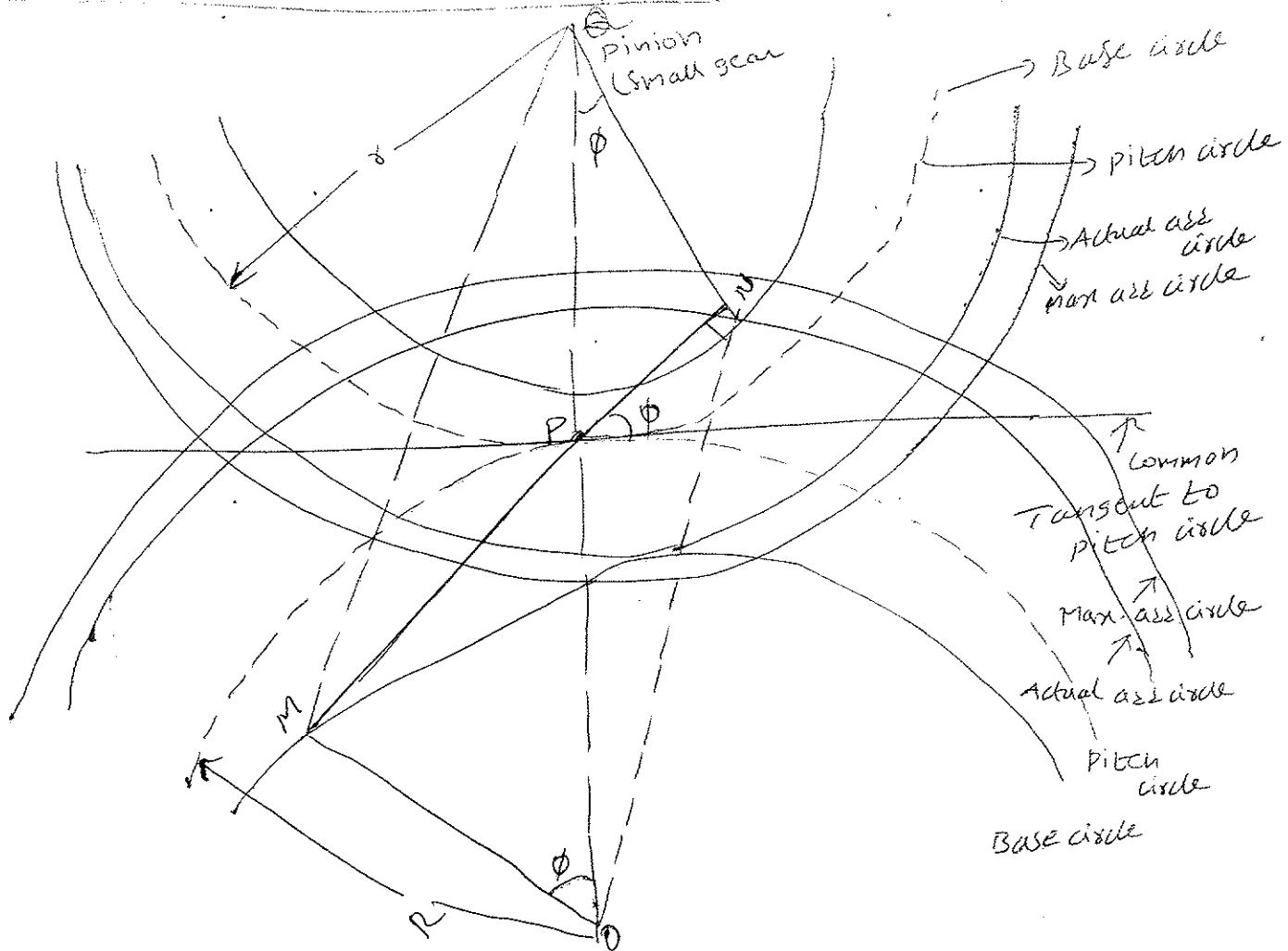
Also, the speed of road wheel attached to axle 'Q' or
the speed of gear 'D' is 210 r.p.m, Therefore from the
fourth row of table,

$$\begin{aligned}y - n &= 210 \quad \text{or} \quad n = y - 210 \\n &= 200 - 210 \\n &= -10\end{aligned}$$

\therefore Speed of road wheel attached to axle 'P'

$$\begin{aligned}&= \text{Speed of gear 'C'} = n + y \\&= -10 + 210 \\&= 190 \text{ r.p.m Ans}\end{aligned}$$

The minimum number of teeth required on the pinion in order to avoid interference. (Expansion Pinions-Gear arrangement)



Let

T = number of teeth on the wheel (or on larger gear)

t = number of teeth on the pinion (or on smaller gear)

m = module of teeth

d = pitch circle diameter of pinion = $m \times t$

γ = pitch circle radius of pinion = $\frac{d}{2} = \frac{m \times t}{2}$

D = pitch circle diameter of the wheel = $m \times T$

R = pitch circle radius of the wheel = $\frac{D}{2} = \frac{m \times T}{2}$

G_r = gear ratio = $\frac{T}{t}$ also = $\frac{R}{\gamma}$.

α = pressure angle or angle of obliquity.

The point Q is the centre of pinion and point O is the centre of wheel.

From triangle MPQ, which is towards pinion side, we have

$$QM^2 = QP^2 + PM^2 - 2QP \times PM \times \cos \angle QPM.$$

$$= r^2 + R^2 \sin^2 \phi - 2rR \sin \phi \times \cos(90^\circ + \phi)$$

$$(\because \angle QP = \gamma, PM = R \sin \phi \text{ and } \angle QPM = 90^\circ + \phi)$$

$$= r^2 + R^2 \sin^2 \phi - 2rR \sin \phi \times (-\sin \phi)$$

$$(\because \cos(90^\circ + \phi) = -\sin \phi)$$

$$= r^2 + R^2 \sin^2 \phi + 2rR \sin^2 \phi$$

$$= r^2 \left[1 + \frac{R^2 \sin^2 \phi}{r^2} + \frac{2rR \sin^2 \phi}{r^2} \right]$$

$$= r^2 \left[1 + \frac{R^2 \sin^2 \phi}{r^2} + \frac{2rR \sin^2 \phi}{r} \right]$$

$$= r^2 \left[1 + \frac{R}{r} \left(\frac{R}{r} + 2 \right) \sin^2 \phi \right]$$

$$\theta_M = \gamma \sqrt{1 + \frac{R}{r} \left(\frac{R}{r} + 2 \right) \sin^2 \phi}$$

$$= \frac{m \times t}{2} \sqrt{1 + \frac{R}{r} \left(\frac{R}{r} + 2 \right) \sin^2 \phi}$$

$$= \frac{m \times t}{2} \sqrt{1 + \frac{T}{t} \left(\frac{T}{t} + 2 \right) \sin^2 \phi} \quad (\because \frac{R}{r} = \frac{T}{t} = n)$$

now we know that the addendum of the pinion.

$$= \theta_M - \theta_P$$

$$= \theta_M - \gamma \quad (\because \theta_P = \gamma = \frac{m \times t}{2})$$

θ, ϕ, \dots etc.

$$= \frac{m \times t}{2} \int 1 + \frac{t}{E} \left(\frac{t}{E} + 2 \right) \sin^2 \phi = \frac{m \times t}{2}$$

$$= \frac{m \times t}{2} \left[\sqrt{1 + \frac{t}{E} \left(\frac{t}{E} + 2 \right) \sin^2 \phi} - 1 \right] \quad \rightarrow ①$$

Let $A_{P \times m}$ = Addendum of the pinion to avoid interference

where A_P = fraction by which the standard addendums of one module for the pinions should be multiplied in order to avoid interference.

$$\therefore \text{Addendum of pinion} = A_{P \times m}$$

→ ②

Equating the two values of addendum of pinion given by equations (a.18), and ①, we get

$$A_{P \times m} = \frac{m \times t}{2} \left[\sqrt{1 + \frac{t}{E} \left(\frac{t}{E} + 2 \right) \sin^2 \phi} - 1 \right]$$

$$A_P = \frac{t}{2} \left(\sqrt{1 + \frac{t}{E} \left(\frac{t}{E} + 2 \right) \sin^2 \phi} - 1 \right)$$

$$t = \frac{2 A_P}{\left[\sqrt{1 + \frac{t}{E} \left(\frac{t}{E} + 2 \right) \sin^2 \phi} - 1 \right]}$$

$$\therefore t = \frac{2 A_P}{\left[\sqrt{1 + \frac{t}{E} \left(\frac{t}{E} + 2 \right) \sin^2 \phi} - 1 \right]}$$

The above eqn is used for finding the min. number of teeth required in the pinion to avoid inter. (S.M.C.B. unit, Pg. -43/5)

Problems

- Q) Two gears mesh externally and are to give a velocity ratio of 3 to 1. The teeth are of involute form; module 6mm, addendum = one module, pressure angle = 20° . The pinion rotates at 908.p.m. Find:

- No. of teeth on pinion to avoid interference on it and the corresponding number on the wheel.
- The length of path and arc of contact.
- The number of pairs of teeth in contact.
- The max. velocity of sliding.

Sol

Given :

Velocity ratio, $v.r = 3/1$, module, $m = 6\text{mm}$

Addendum = 1 module = $1 \times m = 6\text{mm}$, pressure angle, $\phi = 20^\circ$

Pinion speed = 908.p.m.

\therefore Addendum of pinion or of wheel $= A_p = A_w = 1$
module = 6mm.

- (a) Number of teeth on pinion to avoid interference :

Let:- $T = \text{no. of teeth on wheel}$

$t = \text{no. of teeth on pinion}$.

Now, $v.r = \frac{T}{t} \Rightarrow G = \frac{T}{t} = \frac{3}{1}$ where G = gear ratio

$$\frac{3}{1} = \frac{T}{t}$$

$$T = 3t$$

To avoid interference, the no. of teeth on pinion is given by

Ques.

$$t = \frac{2 \times A_p}{m \cos \phi}$$

where A_p is addendum on pinion = 6 mm

$$= \frac{2 \times 6}{\left[\sqrt{1 + 3(3+2) \sin^2 20^\circ} - 1 \right]} = \frac{12}{\left[\sqrt{1 + 15 \times 0.342^2} - 1 \right]} \\ = \frac{12}{(\sqrt{2.754} - 1)} = \frac{12}{(1.66 - 1)} = \frac{12}{0.66} = 18.2 \text{ says } \underline{19. \text{ Ans}}$$

Ans corresponding number of teeth on wheel

$$T = 3 \times t = 3 \times 19 = \underline{57 \text{ Ans}}$$

(b) Length of path and arc of contact :-

we know that pitch circle radius of pinion

$$r = \frac{m \times t}{2} = \frac{6 \times 19}{2} = 57 \text{ mm}$$

\therefore Radius of addendum circle of pinion,

$$r_a = r + \text{addendum on pinion (A_p)} \\ = 57 + 6 \\ = 63 \text{ mm}$$

now pitch circle radius of wheel

$$r = \frac{m \times T}{2} = \frac{6 \times 57}{2} = 171$$

\therefore Radius of addendum circle on wheel,

$$r_a = r + \text{addendum of wheel (A_w)} \\ r_a = 171 + 6 = 177 \text{ mm}$$

we know that path of approach,

$$= \sqrt{r_a^2 - r^2 \cos^2 20^\circ} - r \sin 20^\circ$$

$$= \sqrt{177^2 - 171^2 \cos^2 20^\circ} - 171 \sin 20^\circ$$

$$= \sqrt{31329 - 25879} - 55.482$$

$$\therefore 14.129 - 55.482 = 15.617 \text{ mm}$$

and path acc²θ.

$$\begin{aligned} &= \sqrt{33^2 + 3^2 \cos^2 \theta - 2 \sin \theta} \\ &= \sqrt{63^2 - 57^2 \cos^2 20^\circ - 57 \sin 20^\circ} \\ &= \sqrt{3969 - 2862 - 19.494} \\ &= \sqrt{1107} - 19.494 \\ &= 33.271 - 19.494 \\ &= 13.777 \text{ mm} \end{aligned}$$

∴ Length of path of contact

$$\begin{aligned} &= \text{path of approach} + \text{path of recession} \\ &= 15.747 + 13.777 \\ &= 29.524 \text{ mm} \quad \text{Ans} \end{aligned}$$

$$\text{Length of arc of contact} = \frac{\text{Length of path of contact}}{\cos \theta}$$

$$\begin{aligned} &= \frac{29.524}{\cos 20^\circ} \\ &= 31.42 \text{ mm} \quad \text{Ans} \end{aligned}$$

(Q) Number of pairs of teeth in contact :-

$$\text{No. of pairs of teeth in contact} = \frac{\text{Length of arc of contact}}{\text{Circular pitch (PC)}}$$

$$= \frac{31.42}{\pi \times 1}$$

$$= \frac{31.42}{\pi \times 6}$$

$$= 1.667(08) \cdot 2 \cdot \cancel{2.5} \quad \text{Ans}$$

(Q) Max. velocity of sliding :-

Let ω_2 = angular speed of wheel in rad/sec

$$\omega_1 = \text{Angular speed of pinion} = \frac{2\pi \times 90}{60} = \frac{2\pi \times 90}{60} = 9.42 \text{ rad/sec}$$

we know that, $\frac{\omega_1}{\omega_2} = \frac{\pi}{t} = 3.0$

$$\omega_2 = \frac{\omega_1}{3} = \frac{9.42}{3} = 3.14 \text{ rad/sec}$$

max. velocity of sliding, is given by,

$$v_s = (\omega_1 + \omega_2) \times \text{path of approach}$$

$$= (9.42 + 3.14) \times 15.747 \quad \left(\begin{array}{l} \text{"path of approach"} \\ \text{"path of recess"} \end{array} \right)$$

$v_s = 197.97 \text{ mm/sec}$ Ans

④ Interference b/w Rack and pinion and minimum number of teeth on a pinion for Involute Rack in order to avoid Interference.

The following Fig show rack and pinion in mesh. The pinion is rotating in clockwise direction and driving the rack.

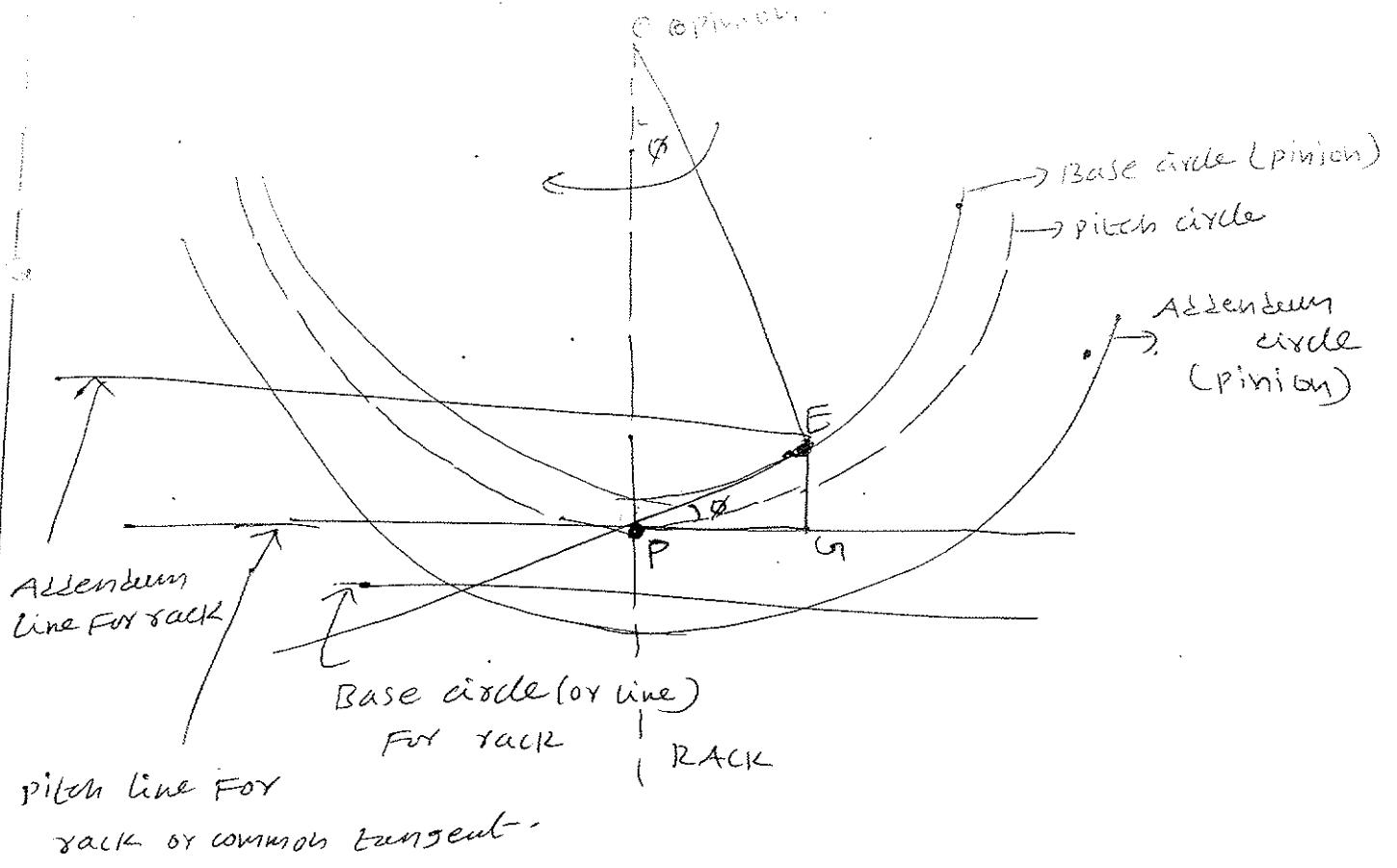
Let, r = pitch circle radius of the pinion

$$= \frac{m \times t}{2}$$

where t = no. of teeth on pinion and m = module

α = pressure angle.

$AR \times m$ = Addendum for rack where AR is the Addendum co-efficient by which the standard addendum of one module for the rack is to be multiplied.



Now

$$GE = PE \sin \phi$$

$$= (r \sin \phi) \sin \phi \quad (\because PE \text{ from right angle triangle } PAE = r \sin \phi)$$

$$= r \sin^2 \phi$$

$$\left(\because r = \frac{m \times t}{2} \right)$$

$$= \frac{m \times t}{2} \sin^2 \phi$$

There will be no interference if,

$$GE \geq AR \times m$$

$$\frac{m \times t}{2} \sin^2 \phi \geq AR \times m$$

$$t \geq \frac{2 AR \times m}{m \times \sin^2 \phi}$$

$$t \geq \frac{2 AR}{\sin^2 \phi}$$

min. no. of teeth on pinion for no interference is given by

$$z_{\min} = \frac{2}{\sin^2 \phi}$$

If $\alpha = 20^\circ$ which is true for Standard Addendum,

$$t_{\min} = \frac{2}{\sin^2 \phi}$$

if pressure angle $\phi = 20^\circ$, Then

$$t_{\min} = \frac{2}{\sin^2 20^\circ} = 17.1 \text{ (or) } 18$$

The min. no of teeth required on the pinion to avoid interference is 18.

A pinion and rack are in mesh. The rack is driven by pinion of 125mm pitch circle diameter. The no. of involute teeth on the pinion are 20. The addendum of both pinion and rack is 6.25mm. If the interference is to be avoided, find the least pressure angle.

Given:- pitch circle diameter of pinion, $d = 125\text{mm}$

$$\therefore \text{pitch circle radius of pinion, } r = \frac{125}{2} = 62.5\text{mm}$$

No. of teeth on pinion, $t = 20$

Addendum of rack, $g_E = 6.25\text{mm}$

Let ϕ = least pressure angle to avoid interference.

Let ϕ = least pressure angle to avoid interference.

We know that addendum of rack is given by eqn.

$$g_E = r \sin^2 \phi$$

$$6.25 = 62.5 \sin^2 \phi$$

$$\sin^2 \phi = \frac{6.25}{62.5} = 0.1$$

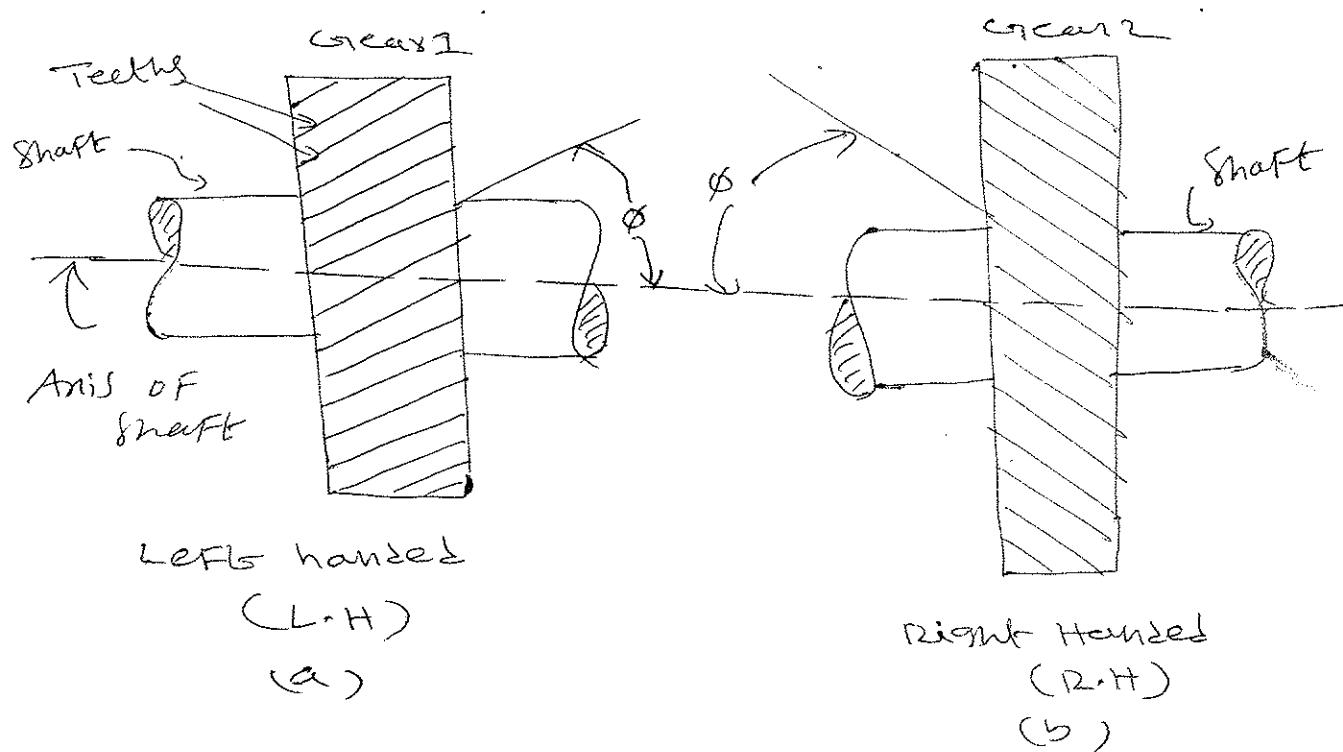
$$\sin \phi = 0.3162$$

$$\phi = \sin^{-1} 0.3162$$

$$\therefore \phi = 18.435^\circ \quad \text{Ans.}$$

HELICAL GEARS

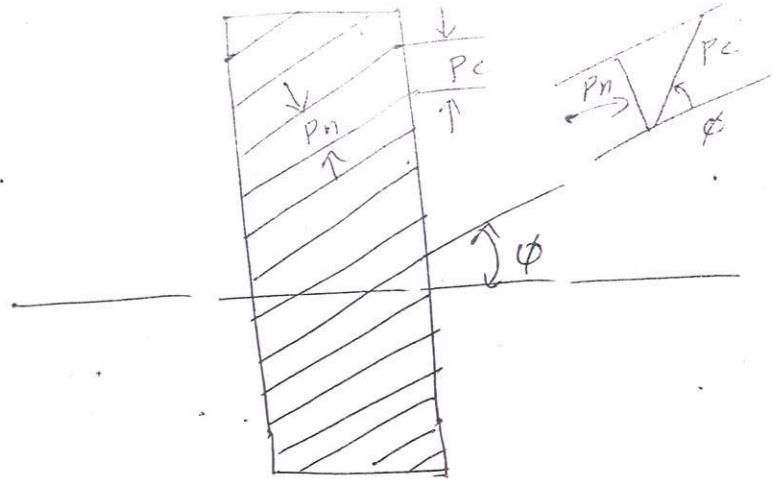
It is already mentioned that two gears used to connect two parallel shafts and having teeth inclined to the axes of shafts are known as Helical gears.



The above Fig shows two helical gears. The gear 1 in Fig(a) is left-handed helical gear, as the helix slopes towards the left of the viewer when this gear is viewed parallel to the Axis of shaft. Hy gear 2 in Fig(b) is a right-handed helical gear.

TERMS IN HELICAL GEARS

- ① Helix angle :- The angle to which the teeth are inclined to the axis of gear, is known as helix angle. It is denoted by ϕ , it is also known as spiral angle of the teeth.



- (2) Normal pitch (P_n) :- The shortest distance b/w similar faces of the adjacent-teeth is known as normal pitch. It is denoted by P_n . The normal pitch of two mating gears must be same.
- (3) circular pitch (P_c) :- The distance measured parallel to the axis b/w similar faces of adjacent teeth is known as a circular pitch. It is denoted by P or P_c . It is also known as axial pitch (or) transverse pitch.

$$\therefore P_n = P_c \cos \phi$$